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"THE PHILOSOPHICAL MAGAZINE" AND THE HISTORY OF MATHEMATICS.

BY GINO LORIA.*

BEFORE reviews devoted to particular branches of science had come into existence, results obtained by individual research were diffused by means of letters to those who were likely to be interested. At a later date they were published in encyclopaedic journals, in literary periodicals, and in the *Proceedings* of the various academies. This makes it extremely difficult to obtain accounts both accurate and complete of much that has been written in this or that particular subject. To assist in removing this difficulty, and to bring within the reach of the student the results of historical research, I have suggested, for instance, an examination of the periodicals, etc., which, while not exclusively devoted to science, may nevertheless contain material of the greatest value to us in the history of science. The suggestion had the honour of receiving the approval of one eminent bibliographer, and, in his turn, he did for three rare German periodicals † what I had done for two of the most celebrated Italian literary journals.‡

The difficulty to which I have referred is met with in its most acute and regrettable form in those investigations of a more or less historical character which are from time to time to be found in publications in which one would never dream of looking for them. It will be thus seen that the carrying of my proposal into effect is a matter of considerable and pressing importance. It so happens that quite recently there came into my hands certain papers on the history of mathematics from the pen of an English writer, who had a certain reputation in his day as a mathematician, but whose historical writings have escaped notice. I refer to ten short articles § entitled *Geometry and*

* Reprinted by kind permission of the author from the *Rivista di Storia Critica delle Scienze Mediche e Naturali.* Year VII. No. 1. Jan.-Feb. 1916. Notes in [ ] are by W. J. G.

† F. Müller, *Die " Bibliothèque Germanique," das " Journal littéraire d'Allemagne " und die " Nouvelle Bibliothèque germanique " als Quellen für die Geschichte der Mathematik in XVIII. Jahrhundert.* (" Festschrift M. Cantor anlässlich seines achzigsten Geburtstags," Leipzig, 1909, pp. 62-77.)

‡ *Il " Giornale de' Letterati d' Italia " di Venezia e la " Raccolta Calogerà " come fonti per la storia delle matematiche nel secolo XVIII.* (*Abh. zur Gesch. der Mathematik*, IX. Heft, 1909, 241-74).

§ I. *Phil. Mag.* XXXII. 1848, 419-21; II. *Id.* XXXIII. 201-06; III. (?) XXXIV. or *Id.* 513-24; IV. *Id.* XXXV. 1849, 497-510; V. *Id.* XXXVI. 1850, 382-94; VI. *Id.* XXXVII. 1850, 198-212; VII. *Id.* IV. Ser. 1. 1851, 536-44; VIII. *Id.* II. 1851, 444-46; IX. *Id.* III. 1852, 286-90; X. *Id.* 523-25 (? *Id.*).

Geometers, which were published, partly after the death of the author, in *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science.*

In these articles the author, Thomas Stephen Davies * gives various notices of English mathematicians of the eighteenth century and their works, his information having been in the main derived from the correspondence of R. Simson, and from unpublished manuscripts. Some of them are solely concerned with particulars of a financial nature connected with mathematical publications, and thus belong rather to the literary history of the past; some are merely corrections of details in the published biographies of scientific men; † but others afford information of real and permanent interest. For instance, here it is reported that in the Escurial Library has been found an Arabic version of the *De Sectione Determinata* of Apollonius Pergaeus. This note has a flavour of novelty about it, and is substantially correct. I shall be surprised if it does not tempt some connoisseur in Islamic literature to publish it with a translation, making it accessible to most students. Such an undertaking would go far to fill a lamentable gap in the history of Greek mathematics.

Of particular interest is the fifth of the articles by Davies, which contains a text, differing from that of Commandinus, extracted from a MS. in the Savilian Library at Oxford, of that part of the *Collectiones Mathematicae* of Pappus Alexandrinus which gives indications of the objects or contents of Euclid's Porisms. The transcription and collation of this manuscript was undertaken by S. P. Rigaud, the well-known Savilian Professor at the University, by whom the work was completed in 1815 or 1816, at the request of Leybourn, the editor of *The Mathematical Repository*. If I am not mistaken, this text has been ignored by all subsequent writers on Pappus and his work.

Another article refers to the endeavours made by Davies to discover the whereabouts of the unpublished works of the celebrated Matthew Stewart—leading to the definite and disappointing conclusion that the manuscripts left by him to his son Dugald had been, perhaps by mutual agreement, consigned to the flames.

There is another article worthy of note, in which Davies deals with a geometer whose merits have been perhaps undeservedly ignored by posterity, that on the volume entitled *Geometrical Amusements, or a Course of Lessons in Construction and Analysis*, by J. H. Swale (Liverpool, 1821). To this volume there should be a sequel, and it will probably be found incorporated in a mathematical periodical entitled *The Liverpool Apollonius, or the Geometrical and Philosophical Repository*, by J. H. Swale, Author of *Geometrical Amusements*, etc., nos. 1-2, 1823-1824, of which Swale was for some time the editor. To give our readers an idea of the contents of this volume, we may state that it contains new and elegant solutions of problems taken from Lorenzo Mascheroni's *Geometria del Compasso*, and also gives material which is fundamental in the following problem: Given in a plane two points and three straight lines; join to the given points a point taken on one of the three lines, so that the joins may make equal angles with the remaining lines." To the informa-

* The biographical and bibliographical information given by Poggendorf (*Handwörterbuch*, t. 1, p. 527) is scanty and incomplete. He is said to have been born about 1794, and to have died on Jan. 6th, 1851. Supplementary information as to Davies and his work may be obtained from the following periodicals, quoted by Sir James Cockle in the *Phil. Mag.* IV. Ser. Vol. I. pp. 536-7, inaccessible to me: *The Mechanic's Magazine*, n. 1431; *The Architect and Civil Engineer*, n. 166; *The Expositor*, n. 18.

† There is a rather curious letter from Maclaurin's son, addressed to Nourse, the publisher of Goldsmith's *Animated Nature*, with the object of contradicting the following particulars therein related of the great commentator of Newton. "For one person to yawn is sufficient to set the rest of the company a-yawning. A ridiculous instance of this was commonly practised upon the famous Maclaurin, one of the professors at Edinburgh. He was very subject to have his jaw dislocated; so that when he opened his mouth wider than ordinary, or when he yawned, he could not shut it again. In the midst of his harangues, therefore, if any of his pupils began to be tired of his lecture, he had only to gape or yawn, and the professor instantly caught the sympathetic affection; so that he thus continued to stand speechless, with his mouth wide open, till his servant, from the next room, was called in to set his jaw again." Maclaurin, the son, characterised this story as absolutely false, and quoted as proof of the contrary the testimony of hundreds of respectable individuals who had gone through the Edinburgh course. There is no mention of the story in Mr. Tweedie's recent biography of Maclaurin (*Mathematical Gazette*, Vol. VIII. pp. 133-151).

tion given by Davies notable additions were made by T. T. Wilkinson,* who added really valuable material from Swale's manuscripts. In them he found a large number of questions in pure geometry treated by the classical methods of the Greeks, suggested to Swale either by the text-books then in use in English schools, or by the mathematical periodicals of the time, *e.g. The Ladies' Diary, The Gentleman's Diary, The Mathematician, The Burrow Diary, Hutton's Miscellany, The Mathematical Repository, The Student, The Mathematical Companion, The Enquirer* and *The Leeds Correspondent.*

But to return to Davies. We may observe that his articles conclude with sundry reflections on the value of the geometrician's geometric method and on the removal of certain obscurities by using some existing works and correcting others. Unfortunately death robbed us of Davies before he published the letters between Jean Bernoulli and Cramer which were in his possession, and the forthcoming appearance of which he had announced. I am not aware that they were published by other hands; if not, a search should be made for the originals, which contain not merely matter of the greatest interest, but may perhaps include important geometrical developments.

We may conclude that in publishing these articles the reviews were taking no unusual step, as from time to time they published historical investigations. It is well to remember that *The Philosophical Magazine* published two important contributions by De Morgan to the knowledge of his day in connection with the celebrated controversy on the discovery of the infinitesimal calculus.† The independence of judgment by which they were informed was so little to the taste of the Royal Society of London, that they were not issued under the auspices of that body, for at that time the worship of Newton had degenerated into a blind belief in the infallibility of its former President. In one of the volumes of the *Phil. Mag.* there is an article by G. Sloane on the authenticity of the *Geometria* of Boethius,‡ containing particulars hitherto unpublished on certain rare MSS. containing the text of this celebrated work.

All this goes to suggest that an examination of the historical papers published in this celebrated English periodical would be useful. It would throw light on much that is obscure and unknown if researches were disinterred of the existence of which people on the other side of the Channel are unaware. I therefore appeal in the name of the history of science that this veil may be lifted.

GINO LORIA.

Genoa, *7th April*, 1915.

[The following notes connected with the *Phil. Mag.* may be of interest.

William Nicholson (1753-1815), after a couple of voyages as midshipman in the East Indian service, tried an attorney's office in London for eight or nine years. A meeting with Josiah Wedgwood led to his taking up a post at Amsterdam as agent for the sale of pottery. After a few years he returned to London, where he fell in with Thomas Holcroft, a man of strange and varied experiences. His father had been a shoemaker, left his trade and set up livery stables, failed and took to the road as a hawker, taking with him his wife and child. The boy obtained a post as stable assistant at Newmarket. Here his spare time was occupied in miscellaneous reading and the study of music, and before long he had a useful acquaintance with French, German, and Italian. In course of time he was usher in a private school, set up for himself as a teacher, and failed, was prompter in a theatre, joined various strolling companies, and began to write and produce plays. As Paris correspondent of the *Morning Herald* his *tour de force* was connected with the famous *Mariage de Figaro.* He attended the performances of Beaumarchais's play until he had the whole by heart, returned to England, and produced it

* *Additions to the late Mr. T. S. Davies's Notes on Geometry and Geometers* (*Phil. Mag.* IV. Ser. Vol. IV. 1852, 28-33, 201-9).

† *On the Additions made to the Second Edition of the Commercium Epistolicum* (*Phil. Mag.* XXXII. 1848, 446-456) and *On the Early History of Infinitesimals in England* (*Id.* IV. Ser. Vol. 4, 1852, 321-330). In the first of the volumes quoted above will be found another article by De Morgan, entitled *An Account of the Speculations of Thomas Wright of Durham,* which deals with an interesting episode in the history of Astronomy.

‡ *On the Connexion of Pope Gerbert with the Geometry of Boethius* (*Phil. Mag.* XXVI. (?) 1850, 529-39).

in 1784 as *The Follies of the Day*. He will be best remembered as the author of *The Road to Ruin*, produced in 1792, and revived eighty-one years later, then running for 118 nights. He was brought to trial for high treason, *quâ* member of the "Society for Constitutional Information," but was discharged. The influence of Holcroft on Nicholson was considerable. The latter assisted his friend in the composition of novels and plays, and began himself to contribute to the lighter periodicals of the time. But his real interests lay in the direction of scientific enquiry, and no doubt he felt that a measure of literary training would be useful to him in the pursuits he had most at heart. In 1781, says the *Encyc. Brit.*, he published the first edition of his *Introduction to Natural Philosophy*, which had an immediate success, supplanting Rowning's *System*, which till then had held the field. Voltaire's *Elemens de la philosophie de Neuton, mis à la portée de tout le monde* was published in 1738, and immediately translated, revised and corrected by John Hanna. The *Encyc. Brit.* states that Nicholson published a translation of Voltaire's *Elements of the Newtonian Philosophy*, but of this there seems to be, curiously enough, no mention in Mr. G. J. Gray's *Bibliography of Newton*. This was not his only translation. In 1790 he published a translation from the third edition of the year before, Fourcroy's *Elements of Natural History and Chemistry, with Alphabetical Comparative View of the Ancient and Modern Names of Chemical Substances, . . . containing Strictures on the History and present State of Chemistry, Observations on the Antiphlogistic Theory, and the New Nomenclature*, etc. 3 vols. 8°. By this time his aptitudes were realised and his energies appreciated by his contemporaries. He was Secretary to the General Chamber of Manufacturers of Great Britain, and took part in the work of the Society for the Encouragement of Naval Architecture. In 1795 he published, in two quarto volumes, *A Dictionary of Chemistry, exhibiting the present State of the Theory and Practice of that Science, its Application to Natural Philosophy, the Processes of Manufactures, Metallurgy, etc., etc., with a Considerable Number of Tables, expressing the Elective Attractions, Specific Gravities, Comparative Heats, Component Parts, Combinations, and other Affections of the Objects of Chemical Research*. This was re-issued in an enlarged form in 1808, and eventually formed the basis of Ure's *Dictionary*. His *First Principles of Chemistry* had appeared in 1788 (according to the *Encyc. Brit.*), and ran to at least three editions. Abraham Bennet, Curate of Wirksworth, the discoverer of the principle of doubling in electricity, by which the difference of potential between two conductors is indefinitely increased (he will be remembered also as the inventor of the gold-leaf electroscope), led Nicholson to contrive an electrostatic machine in which earth conduction is avoided, and an account of this is given in his *Description of an Instrument which, by the turning of a Winch, produces the two States of Electricity without Friction or Communication with the Earth*, pp. 5, 4°, 1788. His *Experiments and Observations on Electricity*, pp. 24, 4°, 1789, contains papers on the Excitation of Electricity, on the Luminous Appearances of Electricity and the Action of Points, and on Compensated Electricity. About 1800 he began to make a name as a public lecturer on scientific topics, and with his friend the great surgeon, Sir Anthony Carlisle, discovered the electrolysis of water by the voltaic current,—an account of this is to be found in Vol. IV. of the *Journal* which he founded in 1797. Among the early papers of interest in this periodical is Blair's paper announcing the discovery of the aplanatic telescope, Bramah's description of his "Press operating by the Action of Water on the Principle of the Hydrostatic Paradox," etc. Dalton, Humphry Davy, Joseph Priestley, and Thomas Young were among the contributors, and useful summaries of English and foreign memoirs formed a feature.

Nicholson invented machines for comb-cutting and file-making, and a cylinder for printing on linen, etc. His areometer is not unknown in some of the laboratories of to-day.

The stages in the history of his journal are as follows :

A Journal of Natural Philosophy, Chemistry, and the Arts. . . . Edited by W. Nicholson. Vol. 1.—New Series, Vol. 36. 4° (8°) London, 1797-1813.

After Vol. 36 of the new series, the *Journal* was united with the *Philosophical Magazine*, and continued as *The Philosophical Magazine and Journal*.

The Philosophical Magazine, Comprehending the various branches of science, the liberal and fine arts (geology), agriculture, manufactures, and commerce. By A. Tilloch. Vol. 1-42. 8°. London, 1798-1813.

The Philosophical Magazine and Journal, etc. By A. Tilloch (R. Taylor). Vol. 43-68. 8°. London, 1814-26.

Annals of Philosophy; or, Magazine of chemistry, mineralogy, mechanics, natural history, agriculture, and the arts. By T. Thomson. Vol. 1.—New Series, Vol. 12. 8°. London, 1813-26.

After Vol. 12 of the new series, the *Annals* was combined with the *Philosophical Magazine.*

The Philosophical Magazine; or, Annals of chemistry, mathematics, astronomy, natural history, and general science. By R. Taylor and R. Phillips. New and united series of the *Philosophical Magazine and Annals of Philosophy.* Vol. 1-11. 8°. London, 1827-32.

The Edinburgh Journal of Science (exhibiting a view of the progress of discovery in natural philosophy, chemistry, mineralogy, geology, botany, zoology, comparative anatomy, practical mechanics, geography, navigation, statistics, antiquities, and the fine and useful arts. Conducted by D. Brewster. Vol. 1.—New Series, Vol. 6. 8°. Edinburgh, 1824-32.

After Vol. 6 of the new series the *Journal* was combined with *The Philosophical Magazine*, and continued as

The London and Edinburgh Philosophical Magazine and Journal of Science. Conducted by D. Brewster, R. Taylor and R. Phillips. Vol. 1-16. 8°. London, 1832-40.

It was then continued as

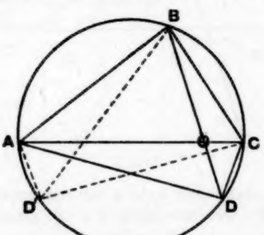
The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science. Conducted by D. Brewster, R. Taylor, R. Phillips, R. Kane, and others. Vol. 17→. 8°. London, 1840→.]

MATHEMATICAL NOTES.

493. [K¹. 8. a.] *Ptolemy's Theorem.*

In the following $\Pi(x, y, \theta)$ denotes the area of a parallelogram of which x and y are two non-parallel sides, and θ one of the angles. It is clear that $\Pi(x, y, \theta)=\Pi(x, y, \pi-\theta)$.

Let $ABCD$ be a quadrilateral inscribed in a circle, and O the intersection of its diagonals. Let α, β, γ, δ be angles at the circumference standing on the arcs AB, BC, CD, and DA respectively; where by the arc AB is meant that arc which does not pass through C and D. It is easily seen that the angle $AOB=\alpha+\gamma$, the angle $BOC=\beta+\delta$, and $\alpha+\beta+\gamma+\delta=\pi$.



(i) Draw through A and C parallels to BD, and through B and D parallels to AC. The resulting parallelogram (omitted in the figure) is $\Pi(AC, BD, \alpha+\gamma)$ or $\Pi(AC, BD, \beta+\delta)$, and is double the given quadrilateral.

(ii) On the arc ADC take the point D', so that $AD'=CD$.

Then the triangles $AD'C$ and CDA will be identically equal, and the quadrilaterals $ABCD'$ and $ABCD$ equal in area.

Draw BD', dividing $ABCD'$ into two triangles.

Since the triangles $AD'C$ and CDA are vertically equal, the angle $CAD'=\delta$.

Hence the angle $BAD'=\beta+\delta$,

and the triangle $BAD'=\frac{1}{2}\Pi(AB, AD', \beta+\delta)$
$=\frac{1}{2}\Pi(AB, CD, \alpha+\gamma)$.

Similarly, the angle $ACD'=\gamma$,

and the triangle $BCD'=\frac{1}{2}\Pi(BC, CD', \alpha+\gamma)$
$=\frac{1}{2}\Pi(BC, AD, \alpha+\gamma)$.

Hence $ABCD'$, and therefore also $ABCD$,

$$=\tfrac{1}{2}\Pi(AB, CD, \alpha+\gamma)+\tfrac{1}{2}\Pi(BC, AD, \alpha+\gamma).$$

(iii) Equating values of double $ABCD$, as given by (i) and (ii),

$$\Pi(AC, BD, \alpha+\gamma)=\Pi(AB, CD, \alpha+\gamma)+\Pi(BC, AD, \alpha+\gamma),$$

from which it easily follows that

$$\text{rect.}\ (AC, BD)=\text{rect.}\ (AB, CD)+\text{rect.}\ (BC, AD).$$

P. J. Harding.

494. [K^1. 6. a.] *De plagis plani infiniti in quibus apparere potest linea tertii ordinis, tribus asymptotis realibus datis praecipuâque satellite.*

[Positum sit in primis $\dfrac{Ax'+By'+C}{A}$ vel affirmativum vel negativum esse quemadmodum punctum (x', y') vel ad dextram vel ad laevam sit lineae $Ax+By+C=0$.]*

Aequatio lineae tertii ordinis asymptotos habentis lineas $x=0$, $y=0$, $lx+my=1$, induet hanc formam

$$xy(lx+my-1)+H(\lambda x+\mu y-1)=0,$$

ubi $\lambda x+\mu y=1$ praecipua satelles est (id est linea in quâ curva asymptotos secat). Duo puncta coincidentia ad infinitionem secundum singulas asymp-

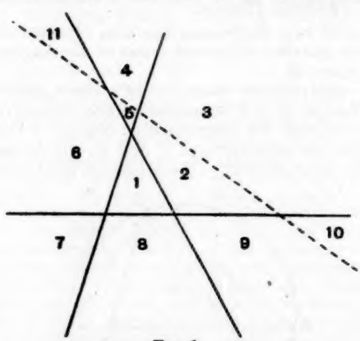


Fig. 1.

totos, valoresque λ, μ, conficiunt octo e novem conditionibus lineae tertii ordinis; et terminus H nonâ constituitur; *e.g.* puncto ad arbitrium capto per quod curva eat.

Ita scribamus aequationem:

$$xy\cdot\frac{lx+my-1}{l}=-\frac{H\lambda}{l}\cdot\frac{\lambda x+\mu y-1}{\lambda}. \quad\text{.....................(i)}$$

* *Math. Gazette*, vol. viii. No. 120, p. 182, Note 460.

Planum asymptotis et lineâ $\lambda x+\mu y=1$ in undecim plagas divisum est. Agamus separatim duo casus in quibus linea $\lambda x+\mu y=1$ (Fig. 1) omnia latera trianguli asymptotôn extrinsecus secat, (Fig. 2), duo latera intus et tertium extrinsecus. Tabulanda sint signa quinque terminorum

$$x,\ y,\ \frac{lx+my-1}{l},\ xy.\frac{lx+my-1}{l},\ \frac{\lambda x+\mu y-1}{\lambda},$$

in singulis plagis.

In plagâ		1	2	3	4	5	6	7	8	9	10	11
x	habet signum	+	+	+	−	−	−	−	+	+	+	−
y	,, ,,	+	+	+	+	+	+	−	−	−	−	+
$\frac{lx+my-1}{l}$	,, ,,	−	+	+	+	+	−	−	−	+	+	−
et igitur												
$xy.\frac{lx+my-1}{l}$	,, ,,	−	+	+	−	−	+	−	+	−	−	+
et $\frac{\lambda x+\mu y-1}{\lambda}$	,, ,,	−	−	+	+	−	−	−	−	−	+	+

Quod ex aequatione (i), quemadmodum terminus $\frac{-H\lambda}{l}$ vel affirmativus vel negativus sit, termini $xy.\frac{lx+my-1}{l}$ et $\frac{\lambda x+\mu y-1}{\lambda}$ signa habent vel eadem

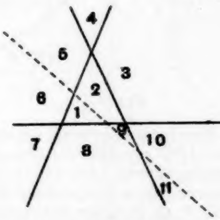


FIG. 2.

vel contraria, ex duobus versibus postremis schematis curva vel in plagis 1, 3, 5, 7, 9, 11, vel in 2, 4, 6, 8, 10 apparet.

Sed in figura secundâ, talia sunt signa :

In plagâ		1	2	3	4	5	6	7	8	9	10	11
x	habet signum	+	+	+	−	−	−	−	+	+	+	+
y	,, ,,	+	+	+	+	+	+	−	−	−	−	−
$\frac{lx+my-1}{l}$	,, ,,	−	−	+	+	−	−	−	−	−	+	+
et igitur												
$xy.\frac{lx+my-1}{l}$	,, ,,	−	−	+	−	+	+	−	+	+	−	−
et $\frac{\lambda x+\mu y-1}{\lambda}$	,, ,,	−	+	+	+	+	−	−	−	+	+	−

et curva vel in plagis 1, 3, 5, 7, 9, 11, vel in 2, 4, 6, 8, 10 apparet.

Iam statuamus regulam generalem de plagis plani in quibus curva apparere potest.

Nota undecim plagas in quibus planum tribus asymptotis et praecipuâ satellite divisum est. Harum una est convexum trapezium, duae sunt trianguli: ex octo residuis plagis infinitis, quas loculos nominabimus, quatuor angulos directo oppositos ad trapezii angulos habent, et quatuor aliae angulos directo oppositos ad angulos triangulorum: curva quidem continetur vel trapezio et ejus directo oppositis loculis, vel duobus triangulis et eorum directo oppositis loculis.

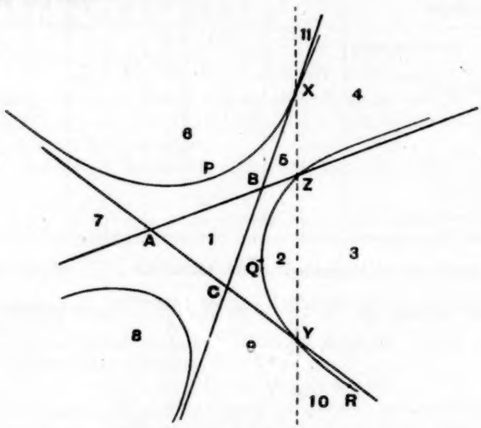


FIG. 3.

Haec regula utilis est in descriptione curvae, asymptotis datis et praecipuâ satellite. Exempli causâ, si ABC asymptotôn triangulus erit (Fig. 3), XYZ praecipua satelles, et Q punctum curvae, linea tertii ordinis apparebit in plagis 2, 4, 6, 8, 10. Descriptio curvae fit e proprietate metricâ: ut, cum quaevis linea PQR curvam in punctis P, Q, R, asymptotosque in X', Y', Z' secat, tum

$$PX'+QY'+RZ'=0.$$

R. W. K. E.

495. [L^1. 14. a.] In the *Mathematical Gazette* for January 1916, No. 121, p. 221, Note 462, Mr. N. M. Gibbins states, and proves analytically, the following interesting property: The square of the major axis of a conic inscribed in a triangle is equal to the sum of the squares of the radii of the director circles of the inscribed conics whose centres are the foci of the original conic.

A semi-geometrical proof may be arranged after this fashion.

Let S, S' be the foci, Ω the centre, and λ, μ the semi-axes of a conic inscribed in the triangle ABC. Then the director circle of any inscribed conic cuts orthogonally the self-polar circle of ABC, whose centre is at the orthocentre H, and whose radius is σ, where $\sigma^2=-4\,\Pi\cos A$.

Since $$SH^2+S'H^2=2\Omega H^2+2\Omega S^2,$$
therefore $$SH^2-\sigma^2+S'H^2-\sigma^2=2(\Omega H^2-\sigma^2)+2\Omega S^2,$$
or $$\rho^2+\rho'^2=2(\lambda^2+\mu^2)+2(\lambda^2-\mu^2)$$
$$=4\lambda^2,$$

ρ, ρ' being the radii of the director circles of the inscribed conics whose centres are at S, S' respectively.

R. F. DAVIS.

496. [C. 2. a, J.] *Squaring the Hyperbola.*

Mr. E. M. Langley's elegant demonstration of the integral of $\sec\theta$, in the *Mathematical Gazette*, May, 1914, p. 335, may be extended so as to "square the hyperbola" at the same time.

In the figure, the line BQU cuts the circle at Q and the straight line OA in U at the same angle; and if displaced into the adjacent position Bqu, crossing NQ in n, $Qq=Qn$,

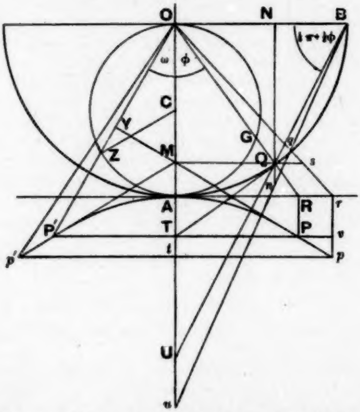
$$\text{(1)} \qquad \sec\phi\, d\phi = \frac{d\phi}{\cos\phi} = \frac{Qq}{NQ} = \frac{Qn}{NQ} = \frac{Uu}{OU} = \frac{dz}{z}, \quad \text{if } OU=z.$$

Then by a definition of the natural logarithm to base e,

$$\text{(2)} \qquad \int_0 \sec\phi\, d\phi = \int_a \frac{dz}{z} = \log_e \frac{z}{a} = \log\frac{OU}{OA},$$

and $OU=OT+TU=a(\sec\phi+\tan\phi)$, or $OU=OB\tan OBU=a\tan(\frac{1}{4}\pi+\frac{1}{2}\phi)$.

This is Mr. Langley's proof.



In the hyperbola AP, the horizontal ordinate $y=TP=TQ=TU=AR$ to abscissa $OT=x$; and while TQ is the tangent of the circle at Q, the tangent of the hyperbola at P is MP, and at right angles to OP'. Then

$$\text{(3)} \qquad \frac{\text{hyperbolic sector element } OPp}{\text{circular sector element } OQq} = \frac{OY\,.\,Pp}{OQ\,.\,Qq} = \frac{OM\,.\,Pv}{OM\,.\,Qs} = \frac{Rr}{Qs} = \frac{OA}{OM} = \frac{1}{\cos\phi};$$

$$\text{(4)} \qquad \text{the sector element } OPp = \tfrac{1}{2}OT\,.\,Qq = \tfrac{1}{2}a^2\sec\phi\, d\phi;$$

and then, by integration, the hyperbolic sector OPP' is a^2 times the integral of $\sec\phi$ in (2), and denoting it by u,

$$\text{(5)} \qquad u = \int_0 \sec\phi\, d\phi = \log(\sec\phi+\tan\phi) = \log\tan(\tfrac{1}{4}\pi+\tfrac{1}{2}\phi)$$
$$= \log\frac{x+y}{a} = \log\frac{a}{x-y},$$

and introducing the notation of the hyperbolic function,

$$\text{(6)} \qquad \operatorname{sh} u = \tan\phi = \frac{y}{a}, \quad \operatorname{ch} u = \sec\phi = \frac{x}{a}, \quad e^u = \tan(\tfrac{1}{4}\pi+\tfrac{1}{2}\phi).$$

The line potential at O of AR, or MQ, or any horizontal ordinate from OA to the line OR is given also by u; while the attraction at O by the line AR or the arc AQ is the same.

Brewster quotes from Newton's commonplace book, July 4, 1699: "At such time I found the method of Infinite Series; and in summer 1665, forced from Cambridge by the plague, I computed the area of the Hyperbola at Boothby Lincolnshire to two and fifty figures by the same method."

This refers presumably to a calculation of e, given after to 10 places in the *Principia*, as Squaring the Circle meant the calculation of π; and Newton's attention about the same time must have been first directed to the idea of Gravitation, according to the legend of the apple.

When u is converted from radians to minutes of angle by multiplying by $180\times 60\div\pi=3438$, the result is called the meridional part (M.P.) of ϕ the latitude, so that

(7) $$\text{M.P.}=3438\log_e\tan(45°+\tfrac{1}{2}\text{ latitude})=7916\log_{10}\tan\tfrac{1}{2}(90°+\text{latitude}),$$

and the M.P. is tabulated to every minute of the latitude for use in Navigation and Geography, on the chart of Mercator projection.

The table of M.P. was first calculated by Edward Wright about 1594, and published in his *Errors of Navigation*, 1599. Wright gave here the exact theory of the construction of the Mercator projection, whereas before in Mercator's map of the world, 1569, the degrees of latitude had been laid off empirically.

Later, in 1616, Wright was the translator into English of the Latin of Napier's Canon, and so associated with the early history of the logarithm. Wright's table of the M.P. was obtained by the continued summation of the secant of the angles, proceeding by increments of one minute, as no table of logarithms was then in existence; so that actually he had calculated a table of the logarithm of the tangent, without being aware of it at the time; and the subsequent identification is related in Benjamin Martin's *Mariner's Mirror*, 1772, where they are called nautical logarithms, and discussed by Florian Cajori in the *Napier Tercentenary Memorial Volume*, 1915.

The conformal representation of the (x, y) stereographic projection, with Ox to the pole, and the (u, v) Mercator chart, is discussed in my *Calculus*, Chapter VI, by means of the relation

(8) $$x+iy=c\operatorname{th}\tfrac{1}{2}(u+iv),\text{ or } c\operatorname{th}\tfrac{1}{2}(u+a+iv)$$

in a linear transformation, v denoting in radians the longitude, and u the M.P. of the latitude.

The transverse axis of the hyperbola has been drawn vertical in the figure, to make it serve in the calculation of the motion of a pendulum, represented by its centre of oscillation G, projected from A along the circle on the diameter AO, with velocity to carry it up to O, and no further.

Then if π/p denotes the beat in seconds of an invisible oscillation at A, and we put $u=pt$ at a subsequent time of t seconds, when the pendulum has advanced through an angle θ, we find

(9) $$\phi=\tfrac{1}{2}\theta,\quad \tan\tfrac{1}{2}\theta=\operatorname{sh}pt,\quad \sec\tfrac{1}{2}\theta=\operatorname{ch}pt,\quad \frac{d\theta}{dt}=2p\operatorname{sech}pt,$$

so that the motion is expressed by the hyperbolic function and the M.P.

If the half beat of the pendulum in invisible oscillation is taken as the unit of time interval, and the corresponding M.P. in minutes, the angle ϕ or $\frac{1}{2}\theta$ is read off in degrees and minutes in the table of M.P.

pt	$\frac{1}{2}\pi$	π	$\frac{3}{2}\pi$	2π	$\frac{\frac{1}{2}\pi}{2}$	$\frac{\frac{1}{2}\pi}{10}$
M.P.	5400	10800	16200	21600	2700	540
ϕ	66° 30′	85° 3′	89°	89° 47′	40° 59′	8° 57′
θ	123°	170° 6′	178°	179° 34′	81° 58′	17° 54′

Thus in two beats the pendulum will have reached within half a degree of the highest point O.

The line CZ oscillates about C like a magnet, disturbed from the magnetic meridian OA with velocity just sufficient to carry it away through a right angle into the position of unstable equilibrium, and then

$$(10)\quad \tan\omega=\frac{TP}{OT}=\frac{TQ}{OT}=\sin\phi=\operatorname{th}pt,\quad \tan 2\omega=\operatorname{sh}2pt,\quad \frac{d\omega}{dt}=p\cos 2\omega.$$

Sept. 14, 1916. G. Greenhill.

497. [R. 8. d.] *On the Compound Pendulum.*

The pendulum should be a rod (as uniform as possible) and one sliding piece, carrying a knife-edge, which can be fixed in any position by a screw. The sliding piece is moved down the rod to a position such that the periodic time about the knife-edge which it carries is a minimum. The sliding piece oscillates with the rod.

If m be the mass of the rod,

m_1 the mass of the sliding piece and knife-edge,

h the distance of the C.G. of the *rod* from the knife-edge,

h_1 the distance of the C.G. of the sliding piece from the knife-edge,

k the radius of gyration of the rod about an axis through *its* C.G. parallel to the knife-edge,

k_1 the radius of gyration of the sliding piece about the knife-edge,

then the periodic time $\quad t=2\pi\sqrt{\dfrac{m(k^2+h^2)+m_1k_1^2}{g(mh+m_1h_1)}}.$

h is the only variable in this formula for t, and t is a minimum when

$$m(k^2+h^2)+m_1k_1^2=(mh+m_1h_1)2h.$$

Hence for the minimum periodic time, $t=2\pi\sqrt{\dfrac{2h}{g}}.$

This formula is exactly the same as that which is obtained when the knife-edge carrier does not oscillate, but it should be noticed that h is not equal to k in the above. The difficulty of finding the C.G. of the rod can be overcome by inverting the rod and using the same knife-edge carrier to find the minimum periodic time for the inverted rod.

If T and H are the new corresponding values of t and h,

$$T=2\pi\sqrt{\frac{2H}{g}};$$

$$\therefore\quad t^2+T^2=\frac{8\pi^2}{g}(h+H).$$

$h+H$, being the distance between the two positions of the knife-edge, can be easily determined.

Thus the difficulty of finding the C.G. is overcome, as in Kater's principle, while the method is much simpler in application.

Also, since the knife-edge carrier itself oscillates, the rod does not need to be specially constructed.

The advantage of the minimum-period method is that g is independent of a small error in h.

For $\quad t=2\pi\sqrt{\dfrac{m(k^2+h^2)+m_1k_1^2}{g(mh+m_1h_1)}}=2\pi\sqrt{x}$, say;

$$\therefore\quad g=\frac{4\pi^2.x}{t^2};$$

$$\therefore\quad \delta g=\frac{\partial g}{\partial t}\delta t+\frac{\partial g}{\partial h}\delta h=-\frac{8\pi^2}{t^3}x\,\delta t+\frac{4\pi^2}{t^2}\frac{\partial x}{\partial h}\delta h.$$

But $\dfrac{\partial x}{\partial h}=0$, being the condition that t is a minimum;

$\therefore\ \delta g$ is independent of a small error in h. A. W. Lucy.

498. [K[1]. 6. a.] *Note on the Area of a Triangle in Plane Co-ordinate Geometry.*

The determinant form of the expression for the area of a plane triangle in Cartesian rectangular co-ordinates is well known. The object of this note is to show that when the vertices of the triangle are given in polar co-ordinates, the area can also be expressed in the form of a determinant.

As given in the text-books, the area is $\frac{1}{2}\Sigma r_1 r_2 \sin(\theta_1-\theta_2)$, *i.e.*

$$\tfrac{1}{2}r_1r_2r_3\Sigma\{\sin(\theta_1-\theta_2)/r_3\} \quad\text{or}\quad \tfrac{1}{2}r_1r_2r_3\,|\,r_1^{-1},\ \sin\theta_1,\ \cos\theta_1\,|,$$

where for reasons of space the first row of the determinant alone is given, both here and after.

It follows that the condition for three collinear points is that the above determinant vanishes.

In proving the property of *Simson's Line* (cf. Smith's *Conic Sections*, 1910, p. 97: ex. 4), the three points whose collinearity is to be shown have co-ordinates $(2a\cos\alpha\cos\beta,\ \alpha+\beta)$, etc. These points can be shown to be collinear without actually finding the equation of the line. Thus, we have for the determinant

$$|\,1/(2a\cos\alpha\cos\beta),\ \sin(\alpha+\beta),\ \cos(\alpha+\beta)\,|,$$
$$1/2a\Pi\cos\alpha\times|\cos\gamma,\ \sin(\alpha+\beta),\ \cos(\alpha+\beta)\,|.$$

To facilitate reduction, let $\alpha+\beta=x$, $\beta+\gamma=y$, $\gamma+\alpha=z$, $\alpha+\beta+\gamma=u$. Then the determinant becomes

$$|\cos(u-x),\ \sin x,\ \cos x\,|$$
$$=\Sigma\cos(u-x)\sin(y-z)=\Sigma\cos\gamma\sin(\beta-\alpha)=0.$$

The condition derived above for the collinearity of three points can also be obtained from the polar equation of the straight line. Thus the line joining the points $(r_1,\ \theta_1)$ and $(r_2,\ \theta_2)$ is

$$\Sigma rr_1\sin(\theta-\theta_1)=0 \quad\text{or}\quad |\,r^{-1},\ \cos\theta,\ \sin\theta\,|=0,^*$$

where r, θ, are current co-ordinates. If this line pass through a third point $(r_3,\ \theta_3)$, we have $\quad |\,r_1^{-1},\ \sin\theta_1,\ \cos\theta_1\,|=0.$

New York City, U.S.A., July 15, 1915. CHARLES N. SCHMALL, B.A.

499. [A. 2. b.] Solve the equation $1+x^4=7(1+x)^4$.

Put
$$1=r\cos\phi,$$
$$x=r\sin\phi,$$
so that
$$1+x^2=r^2,$$
$$x=r^2\cos\phi\sin\phi.$$

Equation is
$$r^4\cos^4\phi+r^4\sin^4\phi=7(r\cos\phi+r\sin\phi)^4,$$
leading to
$$30\cos^2\phi\sin^2\phi+28\cos\phi\sin\phi+6=0,$$
and giving
$$\cos\phi\sin\phi=-\tfrac{3}{5},\ -\tfrac{1}{3}.$$
$$\therefore\ x=-\tfrac{3}{5}r^2 \text{ or } -\tfrac{1}{3}r^2,$$
i.e.
$$1+x^2=-\tfrac{5}{3}x \text{ or } -3x.$$

Thus the four values of x are found.

The method may be adopted to solve equations of the type

$$\begin{cases}(x^4+y^4)(x^2+y^2)=a^6,\\ \qquad\qquad x^6+y^6=b^6,\end{cases}$$

by putting
$$x=r\cos\phi,$$
$$y=r\sin\phi,$$

and proceeding on the above lines. H. FREEMAN.

* This determinant was first given, as far as he is aware, in the writer's *First Course in Analytical Geometry* (Blackie & Son), p. 66, § 40.

REVIEWS.

Russia and Elementary Mathematics.

A new journal devoted to elementary mathematics is published in Russian at Revel, under the title *Mathematical Pages*. It appears ten times yearly, the months of June and July being excepted, and the subscription is 1 rouble 40 kopecks, or for a single number, 20 kopecks. The editor is N. Agromonoff, and the address for communications and subscriptions is: Revel, Luizental-skaya Street. 14. 1.

The journal commenced in 1915, and contains mathematical notes and problems in trigonometry, geometry, and algebra. We offer the cordial greetings of the *Mathematical Gazette* to our new contemporary, with the wish that it may have a prosperous career, and serve as a link in the scientific fellowship between Russia and Great Britain. C. S. J.

Quartic Surfaces with Singular Points. By C. M. Jessop, M.A. Pp. 195. 12s. net. 1916. (Cambridge University Press.)

This important work is a gain to geometry, and should take its place among the classic English treatises. It contains a complete study of a definite section of algebraic geometry, and gives a large number of very interesting facts in a book of moderate size; though this is done by condensing the arguments so much that the result is not easy reading. Over a hundred different types of quartic surfaces are discussed; the properties studied have mainly to do with the curves of orders 1 to 6 lying on the surfaces. The apparatus used ranges from simple geometrical intuitions to double theta functions; differential geometry is almost but not quite excluded. A welcome feature is the Introduction of twenty-five pages, giving a summary of the results of the following chapters.

Chapter I. deals with isolated singularities, though the case which is simplest on a quartic surface, a triple point, is postponed to Chapter VII. A systematic treatment is given when the number of nodes varies from 7 to 16, based on the behaviour of the sextic tangent cone drawn to the surface from a node. Incidentally there are given some unfamiliar properties of plane sextics. Chapter II. is devoted to a particular type, the desmic surface, with 12 nodes, arising in connection with the intersections of edges and faces of desmic tetrahedra. Chapter III. treats of a double conic (without further singularities), and Chapter IV. of a *nodal* conic and also isolated nodes. (This is indeed to copy Salmon's style.) The method of mapping on a plane occurs frequently; some of the properties of this section might perhaps have been obtained more simply by a quadratic transformation of the quartic surface into a quadric or cubic surface, a method used later in the book in connection with Steiner's surface. But there is no mention of the way in which the double and simple lines of the surface make such Cremona transformations possible. A list of seventy-seven types of quartic surfaces with a double conic is given, based on the theory of Elementary Factors, the surface being regarded as the projection of the intersection of two quadratic varieties in four dimensions. The next chapter is again devoted to a special class of surfaces, cyclides. Confocals are treated so as to show their marked analogy with confocal quadrics; this is the only section where metrical properties are prominent, and the reality of the surface is discussed. But we must be wary with an author who considers that a sphere can be real when its radius is pure imaginary. Chapter VI. goes on to surfaces with a double line, including Plücker's surface. The next section deals with surfaces possessing families of conics, including monoids or surfaces with a triple point, of which Steiner's surface is treated very fully. Chapter VIII. is given to rational surfaces, and the expression of the coordinates in terms of parameters. The proof that there are only three types, besides those mentioned in earlier chapters, is not made as clear as it might be; as often elsewhere in the book, some essential parts of the argument are in a footnote and some are omitted. The last chapter is on determinant surfaces, and is chiefly devoted to the symmetroid and the connected Jacobian.

Any student of these subjects must take Salmon as guide, but there was no need to follow his doubtful grammar and bewildering habit of changing names and notations; the reversal of symbols on p. 166 is quite brutal. We hope that the importance and value of the work will soon call for a reprint, in which such points of style will be improved.

Besides the omissions, announced in the preface, of ruled quartics, and of Kummer's surface on which an English treatise exists, there is no discussion of families of quartics, nor of their intersections; and there is only incidental reference to specialized forms of the singularities. For example, the classification of binodes is quite different on a quartic from what it is on a cubic. The statements on pp. 67, 68 are correct, but might easily suggest a false analogy, for they are quotations from a paragraph in Salmon's *Three Dimensions* which does not apply to quartics: a binode can reduce the class by four without its edge lying on the surface. We should like to see a second volume on quartic surfaces with more complicated singularities, and on other outlying branches of the subject. But probably the author was right in excluding these, as they would have exceeded the limits of his reliable and well-arranged textbook on the central part of the theory of singular quartics.

A Treatise on the Circle and the Sphere. By J. L. Coolidge, Ph.D., Assistant Professor of Mathematics at Harvard University. Pp. 603, with 30 figures. 21s. nett. 1916. (Oxford: Clarendon Press.)

In this treatise, which might almost be called an encyclopedia, "preference is shown to those theorems which are unaltered by inversion, and to those which are as general as possible in their scope. The author has tried to say something about every circle that is known by a recognised name." Accordingly, in the first chapter, on elementary plane geometry, after a preliminary discussion of inversion and of tangency, we have a long section devoted to circles and systems of circles connected with a triangle, the Brocard figures being treated at greatest length. The next chapter, on Cartesian plane geometry, is somewhat surprising: the co-ordinates are either trilinear or tetracyclic. We are plunged into elliptic measurement and isotropics, the latter term being carefully defined sixty pages later: our time-honoured X, Y, Z are conspicuous by their absence. So is the triangle ABC. No doubt, much is gained in system and symmetry by naming the fundamental triangle $A_1A_2A_3$, but it gives an unfamiliar air even to the most familiar properties, and it requires the reader to do without the help of a host of mental associations that would have made the thinking far less arduous. The chapter on constructions deals with two or three problems only, with considerable fulness: the last chapter in two dimensions is entitled "The Tetracyclic Plane," and departs from the plan of the book in that it treats not of circles but of cyclics, defined as loci of the vertices of null circles of quadric circle congruences.

In the next part of the book, which treats of spheres, the author has tried to keep as far as possible parallel to the first part, though he is careful to point out the limitations of the parallelism; this endeavour has perhaps made the sphere less attractive than if it had stood entirely on its own merits. There follow chapters on circles and sphere transformations, and then are introduced the oriented circle and sphere. "The circle and the oriented circle should be considered as essentially dissimilar figures; the former is a locus of points, the latter, in the plane, is best handled as an envelope of oriented lines, and considered under a totally different group." The remainder of the book deals with systems of circles, treated from a great many different points of view. An interesting feature is the introduction of the circle cross as a single element, consisting of two circles such that every sphere through one is orthogonal to the other. Each chapter ends with suggestions for further research, which the author is most anxious should be undertaken—to judge from his closing words—by other people.

Most certainly it must be acknowledged that he has done his own share. The amount of industry whose results are here presented is prodigious, and it would require corresponding industry to detect any important omission or inaccuracy. One envies Professor Coolidge the leisure, learning and love

of his subject, which alone could have carried such a labour to completion. The work of compilation is immense, going back as it does for centuries before Euclid; and there are also very considerable contributions of original work, particularly in the section on the oriented figures, though the author's claim is almost too modest to be detected.

Professor Coolidge evidently wishes his style to be noticed. It may be less striking to Americans than to English; but we are not used to 'prime' for 'accent,' 'transform' as a noun and 'factor' as a verb: and the general tone is more conversational than is at all usual in Europe. But in each chapter, as he warms to his subject, he drops this mannerism and treats serious topics in suitable language. The use of 'shall be' and 'will be' is noticeable: the future tense, unfortunately too common with us, is rather emphatic than logical. The spelling is, however, mainly English; this is possibly due to the influence of the Clarendon Press, who have spared no pains in the production of the volume. Another point in which the writer departs from custom, is his habit of appending an adjective when he cites an author. To point out inaccuracies may be useful; but to inform us that he finds one famous mathematician 'pleasant' and another 'overrated' has no possible bearing on either the circle or the sphere.

A work of so monumental a character is likely to become and remain the standard for a long period, and the production of so comprehensive an authority is a thing to be proud of for all concerned. We may consider that the treatment of infinity, though accurate, is lacking in breadth and imagination, and we may regret that there is not more space given to the really elementary algebra of the subject; a few more pages would have done no harm in a book which will be referred to rather than read through. It is not only as a collection of beautiful theorems that this work will be valued. The arrangement is just as important, bringing out their lines of demarcation, the fundamental ideas which they involve and their relations to other branches of mathematics such as the theory of groups of transformations or the geometry of many dimensions. The author has fully carried out the high aim he set before himself: "Circles and spheres force themselves upon our notice in all parts of geometrical science. The result . . . is that there is a colossal mass of literature dealing with circles and spheres, the various parts of which have been developed with little reference to one another. . . . The present work is an attempt, perhaps the first, to present a consistent and systematic account of these various theories." H. P. H.

Problems in the Calculus. By D. D. Leib. Pp. xii + 224. $1. 1916. (Ginn & Co.)

In the main this will probably be most useful to teachers over here as providing a change from examples of the type with which boys are becoming familiar in our ordinary text-books. Its value to the private student is considerably discounted by the absence, deliberate on the part of the author, of answers to more than a few of each type. Where such a book is in general use, any object the compiler may have in view by omitting the answers is defeated after the first term or so, by which time collections of answers made by those who have been through the mill have a mysterious way of being distributed as required.

Guida Allo Studio della Storia delle Matematiche. By Gino Loria. Pp. xvi + 228. 3 l. 1916. (Milan: Hoepli.)

Prof. Loria has contrived to pack into a small space quite a remarkable amount of information. After a few introductory remarks on the historical method in general, and references to standard classics on the subject—in which he draws due attention to the limits of Montucla, and his *troppo vasto programma*, and, even more severely, to the *innumerevoli modificazioni* that the progress of time had rendered necessary in the case of Cantor—he indicates the resources that lie before the student in the form of periodical literature. The second Part deals with such auxiliaries to investigation as catalogues, biographies, bibliographies, collected editions of the works of individuals, correspondence of famous mathematicians, and monographs on their works

with such advice as to the use of these respective facilities to research as experience has shown necessary and useful. There are not so many misprints and slips in the names of authors and titles of books as usual, but a few are to be seen, *e.g.* Lord Raleigh; I. Todhunter: "History of the mathematical of Probabilities," and the like. Altogether the book is well conceived, a handy pocket volume, well printed, and should enjoy considerable popularity.

CORRESPONDENCE.

THE EDITOR OF THE *Mathematical Gazette*.

DEAR SIR,—On p. 295 of the July number of the *Mathematical Gazette*, "A Non-Mathematical Member" refers apparently to an Accuracy Test-Paper for which I was responsible.

He states (i) that actual division gave the answers to two questions as 0·005249 and 0·1908; (ii) that "an official pronouncement was made that the correct answers to the questions are 0·00524 and 0·190."

His second statement is incorrect; the answers circulated by me were given as 0·00525 and 0·191.—Yours truly, A. W. SIDDONS.

THE LIBRARY.

CHANGE OF ADDRESS.

THE Library is now at 9 Brunswick Square, W.C., the new premises of the Teachers' Guild.

The Librarian acknowledges, with thanks, the gift, by the Clarendon Press, of a copy of *Statics: A First Course*. C. O. Tuckey and W. A. Nayler.

SCARCE BACK NUMBERS.

Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them:

Gazette No. 8 (very important).
A.I.G.T. Report No. 11 (very important).
A.I.G.T. Reports, Nos. 10, 12.

With this number of the *Gazette* is issued *A Supplementary Catalogue of Books, Pamphlets and Periodicals*. The latter can be now borrowed by members under the usual conditions.

Sir George Greenhill has presented a complete set of the *Bulletin of the American Mathematical Society*, and the thanks of the Association are due to him as well as to the many other donors whose gifts have been acknowledged from time to time in the *Gazette*.

ERRATA.

Cancel first erratum, p. 324.

viii. p. 257, lines 5 and 6 from foot of page; *for* 23·16 *read* 23·2.

line 4 from foot of page; *for* $\frac{23\cdot1\dot{6}}{10^3}$ *read* $\frac{23\cdot2}{10^3}x$.

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